On the Occurrence of a Phase Transition in Atomic Systems

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Summary

- Overview of the ideas on Quantum Coherence and Coherence Domains
- Coherent States: a simplified approach
- The effect of temperature
- Coherent Interactions

Condensed Matter

- How can a system of weakly interacting atoms organize itself and form highly ordered structures over large scales?

Examples:

Superfluidity/Superconductivity

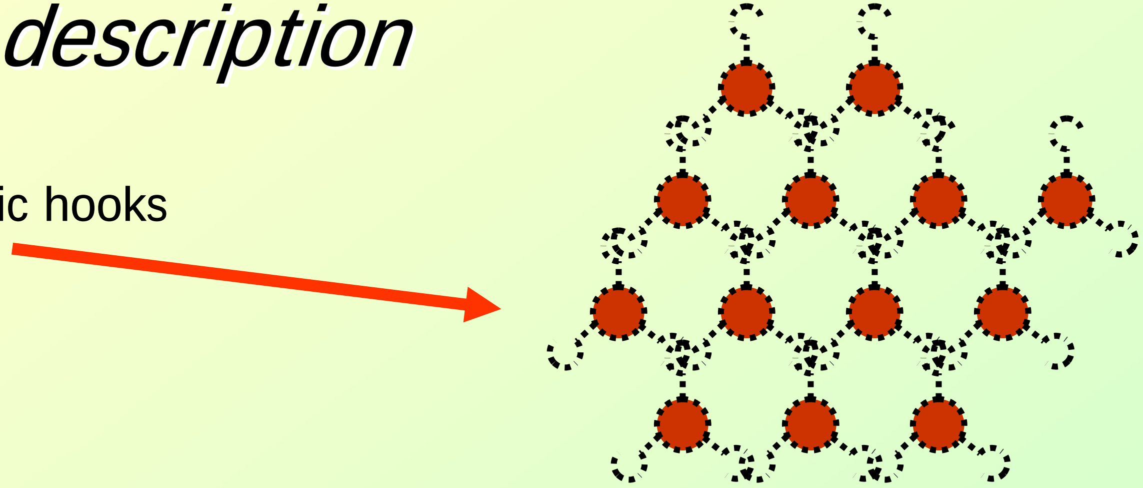
Biological Systems

Crystals

Gas/Liquid/Solid Phase

"Orthodox" description

Paradigm of electrostatic hooks



Question: Is the interaction between neighbors *sufficient* to guarantee long-range order?

What happens if we take into account the radiation field?

Contribution of order N

NEGATIVE!

$$H_{\text{total}} = H^{(0)}_{\text{matter}} + H_{\text{SR}} + H^{(1)}_{\text{rad}} + H^{(2)}_{\text{rad}} + H_{\text{em}}$$

Matter Field

Short-Range

Matter-Field Interaction
(usually neglected)

A^2-term (required by gauge-invariance)

Free e.m. field

A careful analysis shows that COHERENT CONFIGURATIONS exist whose energy is LOWER than those at zero-field

Symmetry Breaking

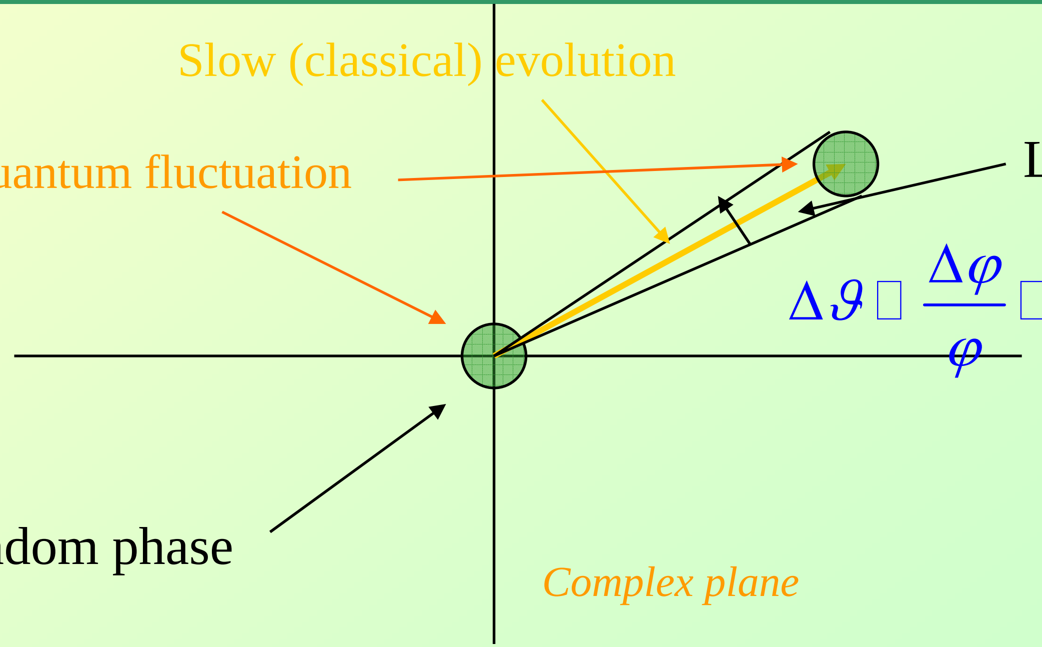
Coherent condensation of a system of *N* identical particles

Not equivalent to Long range!!
Ex: Bose condensate

Definition of quantum coherence

$$\varphi_0\left(\vec{x},\alpha;t\right)=\left\langle\Omega\middle|\Psi\left(\vec{x},\alpha;t\right)\middle|\Omega\right\rangle\neq 0$$

Order parameter



$$\Delta\vartheta \approx \frac{\Delta\varphi}{\varphi} \approx \frac{\Delta\varphi}{\sqrt{N}\,\Delta\varphi}=\frac{1}{\sqrt{N}}$$

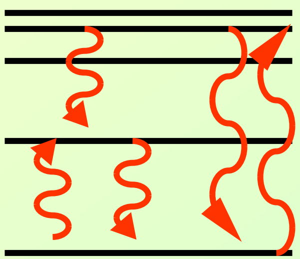
Two-level system

The simplest of the many-body systems

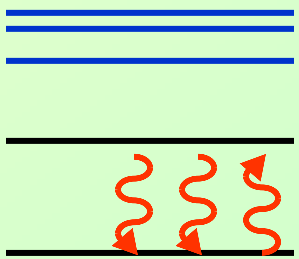
But...

of enormous physical importance

Atomic Energy Levels



Many-level system



Two-level approximation

Coherence equations for the Two-level system

(Preparata Equations)

$$i\dot{\chi}_1(\vec{x},\tau) = g\,\chi_2(\vec{x},\tau)A(\vec{x},\tau)$$

$$i\dot{\chi}_2(\vec{x},\tau) = g\,\chi_1(\vec{x},\tau)A^*(\vec{x},\tau)$$

$$-\frac{1}{2}\ddot{A}(\vec{x},\tau) + i\dot{A}(\vec{x},\tau) - \mu A(\vec{x},\tau) = g\int d^3\vec{y}\,G(\vec{x}-\vec{y})\chi_2^*(\vec{y},\tau)\chi_1(\vec{y},\tau)$$

$$g = eJ\sqrt{\frac{4\pi}{3\omega_0^2}}\sqrt{\frac{N}{V}}$$

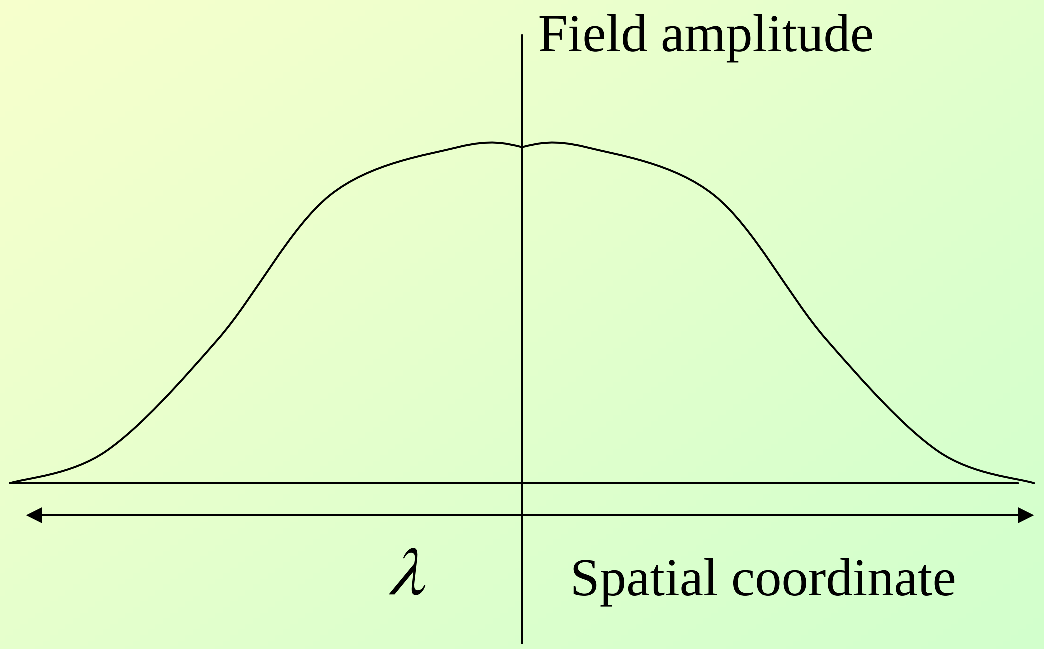
$$\mu = \frac{e^2\lambda}{\omega_0^2}\frac{N}{V}$$

Not present in laser eqs.
Responsible for runaway

Interaction term

$A(\vec{x},\tau)$	Electromagnetic field
$\chi_1(\vec{x},\tau),\ \chi_2(\vec{x},\tau)$	Matter Field

The smallest spatial domain where coherence equations have non-trivial solutions: the Coherence Domain



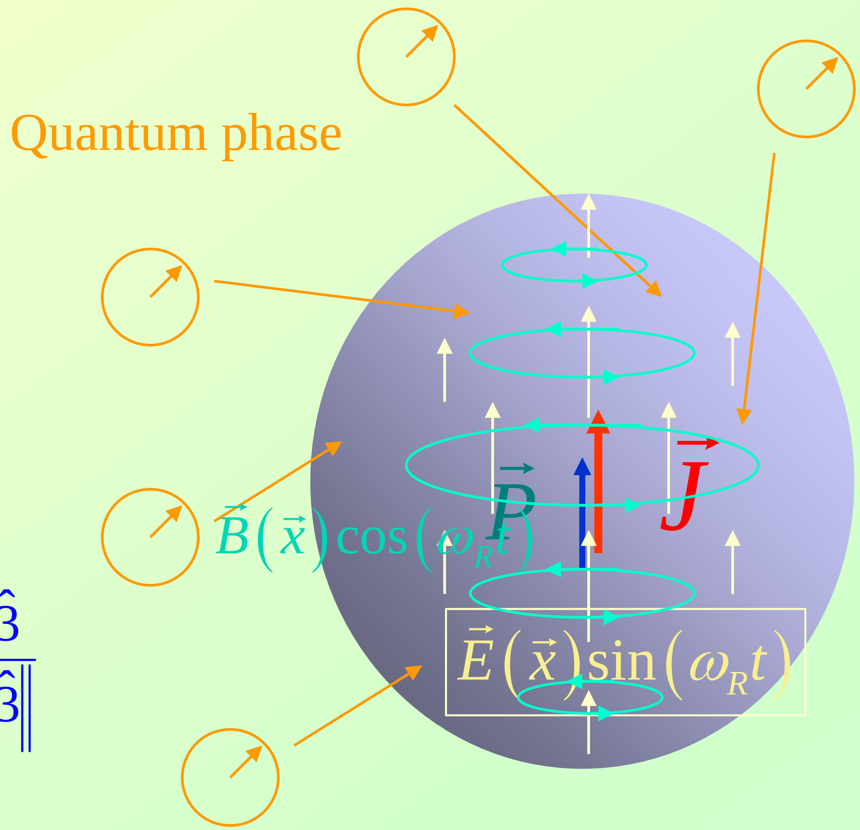
Electromagnetic structure of a single Coherence Domain

The e.m. field has the same phase for each point of the CD

$$\vec{E}(\vec{x},t) = 4\sqrt{\frac{\pi\rho_s}{3\omega_0}}\omega_R A_0\, j_0(\tilde{r})\sin(\omega_R t)\hat{3}$$

$$\vec{B}(\vec{x},t) = 4\sqrt{\frac{\pi\rho_s}{3\omega_0^3}}\sqrt{\frac{6}{5}}A_0\, j_1(\tilde{r})\cos(\omega_R t)\frac{\hat{x}\wedge\hat{3}}{\left\|\hat{x}\wedge\hat{3}\right\|}$$

$$\tilde{r} = \omega_0\sqrt{\frac{6}{5}\left(x_1^2 + x_2^2 + \frac{x_3^2}{2}\right)}$$



Total Reflection of trapped em field

1. CD does not radiate because Poynting vector has zero mean value
2. Due to frequency renormalization, photons cannot radiate because off-shell

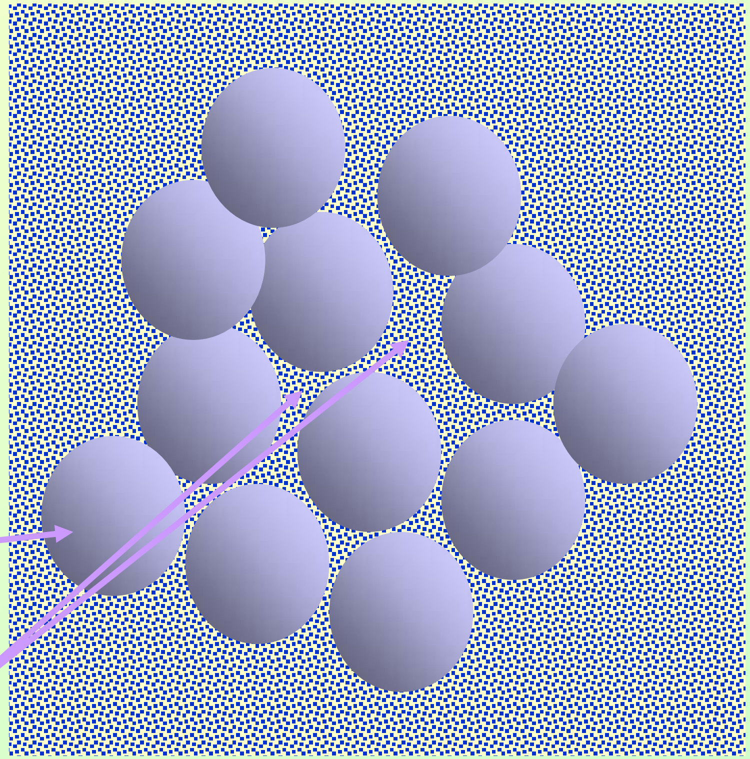
$$\omega_R < \left|\vec{k}\right|$$

Total reflection: a natural 'trapped' laser

Structure of bulk matter

Condensed Matter is viewed as a collection of COHERENCE DOMAINS

2-fluid model



Domain
(Coherent Matter)

Fluctuations
(Incoherent Matter)

Experimental evidence of coherent states at room temperature

Observation of long-ranged many-body attractive forces among sub-micron latex spheres suspended in water, that cannot be explained by means of short-range electrostatic interactions

A.E.Larsen, D.G.Grier, *Nature* 385, 230 (1997)

http://www.mpip-mainz.mpg.de/~deserno/like_charge_attraction.php
http://guava.physics.uiuc.edu/~nigel/courses/569/Essays_2004/files/lu.pdf
http://chronicle.uchicago.edu/970206/colloids.shtml
http://www.mpip-mainz.mpg.de/~deserno/talks/lyon5.ppt
(look at the conclusions)

Superradiance of quantum dots, ***Nature Physics*** **3**, 106 - 110 (2007), M.Scheibner et al.

http://www.nature.com/nphys/journal/v3/n2/abs/nphys494.html

Observation of long range order

Coherent states: a simplified approach

$$H = H_F + H_A$$

$$H_F = \hbar\omega \sum_{s=1,2} c_s^\dagger c_s$$

$$H_A = \frac{1}{2M}\sum_{i=1}^{N}\left(\vec{P}_i - \frac{Ze}{c}\vec{A}(\vec{R}_i)\right)^2 + \frac{1}{2m}\sum_{i=1}^{N}\sum_{j=1}^{Z}\left(\vec{p}_{ij} + \frac{e}{c}\vec{A}(\vec{r}_{ij})\right)^2 + V\left(\vec{R}_i, \vec{r}_{ij}\right)$$

$$\tilde{H} = \sum_{i=1}^{N}\left\{\left[\frac{\vec{P}_i^2}{2M} + \sum_{j\neq i}^{N}\tilde{V}_{ij}\right] + \sum_{z=1}^{Z}\left[\frac{\vec{p}_{iz}^2}{2m} + v\left(\vec{\rho}_{i1}, \vec{\rho}_{i2,\ldots,}\vec{\rho}_{iZ}\right)\right] + \vec{\mu}_i \cdot \vec{E}\left(\vec{R}_i\right)\right\} + H_F,$$

S. Sivasubramanian, A. Widom, and Y. N. Srivastava, Physica A **301**, 241 (2001).

$$\vec{\mu}_i = e\sum_{z=1}^{Z}\vec{\rho}_{iz}$$

$$\vec{E}(\vec{R}) = i\left(\frac{2\pi\hbar\rho\omega}{N}\right)^{1/2}\sum_{s=1,2}\left[\vec{\varepsilon}_s e^{+i\vec{k}\cdot\vec{R}}c_s - \vec{\varepsilon}_s^* e^{-i\vec{k}\cdot\vec{R}}c_s^\dagger\right]$$

The trial Quantum State

EM Glauber state

$$c_s\left|\sqrt{N}\alpha\right\rangle_{em} = \delta_{s1}\sqrt{N}\alpha\left|\sqrt{N}\alpha\right\rangle_{em}$$

Matter field Anzats

$$\psi_\delta\left(\vec{R},\vec{\rho}_1,\vec{\rho}_2,...,\vec{\rho}_Z\right) = \qquad 0\le\delta\le\frac{\pi}{2}$$

$$= i\left\langle\vec{\rho}_1,\vec{\rho}_2,...,\vec{\rho}_Z\middle|\varphi_p\right\rangle\sin\delta\frac{e^{-i\frac{\vec{k}\cdot\vec{R}}{2}}}{\sqrt{V}} + \left\langle\vec{\rho}_1,\vec{\rho}_2,...,\vec{\rho}_Z\middle|\varphi_s\right\rangle\cos\delta\frac{e^{i\frac{\vec{k}\cdot\vec{R}}{2}}}{\sqrt{V}}$$

$$\mu_0\vec{\varepsilon}_1 = \left\langle\varphi_p\middle|\vec{\mu}\middle|\varphi_s\right\rangle \ne 0, \quad \mu_0 > 0$$

Full variational quantum state

$$\left|\Omega(\alpha,\delta)\right\rangle = \left|\sqrt{N}\alpha\right\rangle_{em}\int\prod_{i=1}^{N}\psi_\delta\left(\vec{R}_i,\vec{\rho}_{i1},\vec{\rho}_{i2},...,\vec{\rho}_{iZ}\right)d^3\vec{R}_i\left|\vec{R}_i\right\rangle\prod_{z=1}^{Z}d^3\vec{\rho}_{iz}\left|\vec{\rho}_{iz}\right\rangle$$

Evaluation of the energy-per-particle

$$\frac{E(\alpha,\delta)}{N} = \frac{1}{N}\left\langle \Omega(\alpha,\delta) \middle| \tilde{H} \middle| \Omega(\alpha,\delta) \right\rangle = \frac{\hbar^2 k^2}{8M} + E_s + \hbar\omega\left(\sin^2\delta + \alpha^2 - \gamma\alpha\sin 2\delta\right)$$

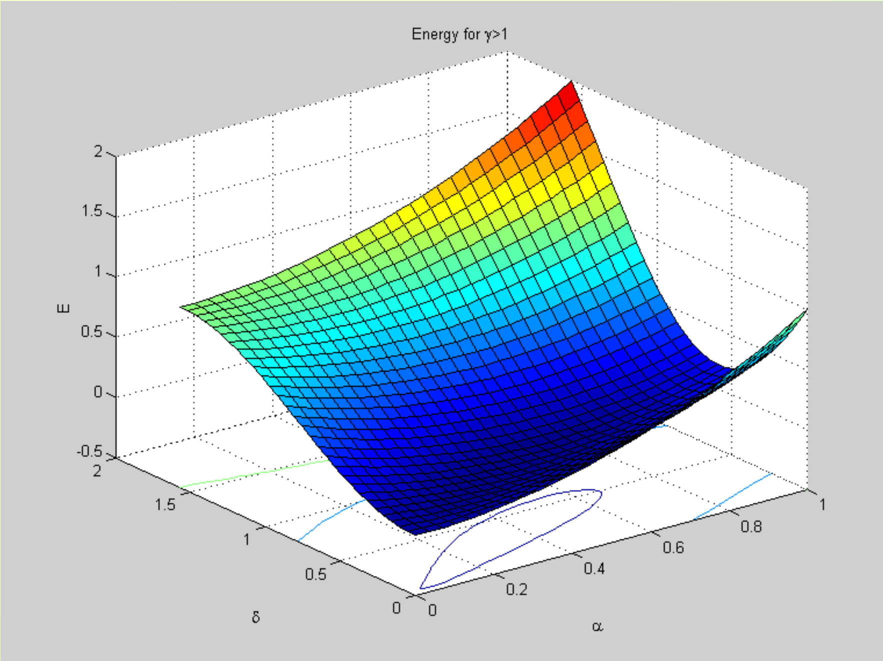
$$\hbar\omega = E_p - E_s \qquad \gamma = \mu_0\sqrt{\frac{2\pi\rho}{\hbar\omega}}$$

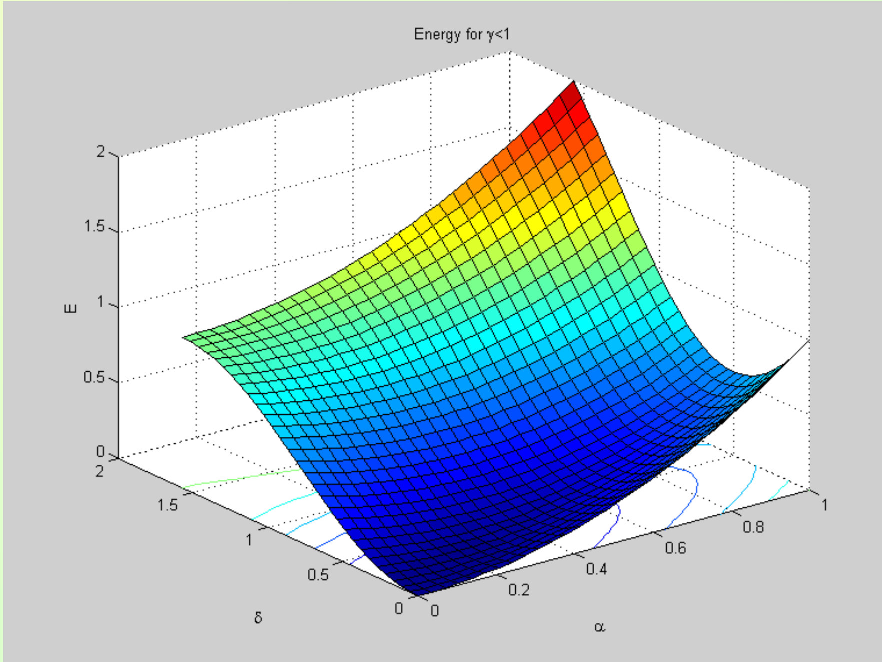
Minimum exists when $\gamma^2 > 1$ $\qquad \hbar\omega << Mc^2$

$$\begin{cases} \dfrac{\partial E}{\partial \alpha} = 0 \\ \dfrac{\partial E}{\partial \delta} = 0 \end{cases} \Rightarrow \quad E_{\min} = E_s + \frac{\hbar\omega}{4}\left(2 - \gamma^2 - \gamma^{-2}\right)$$

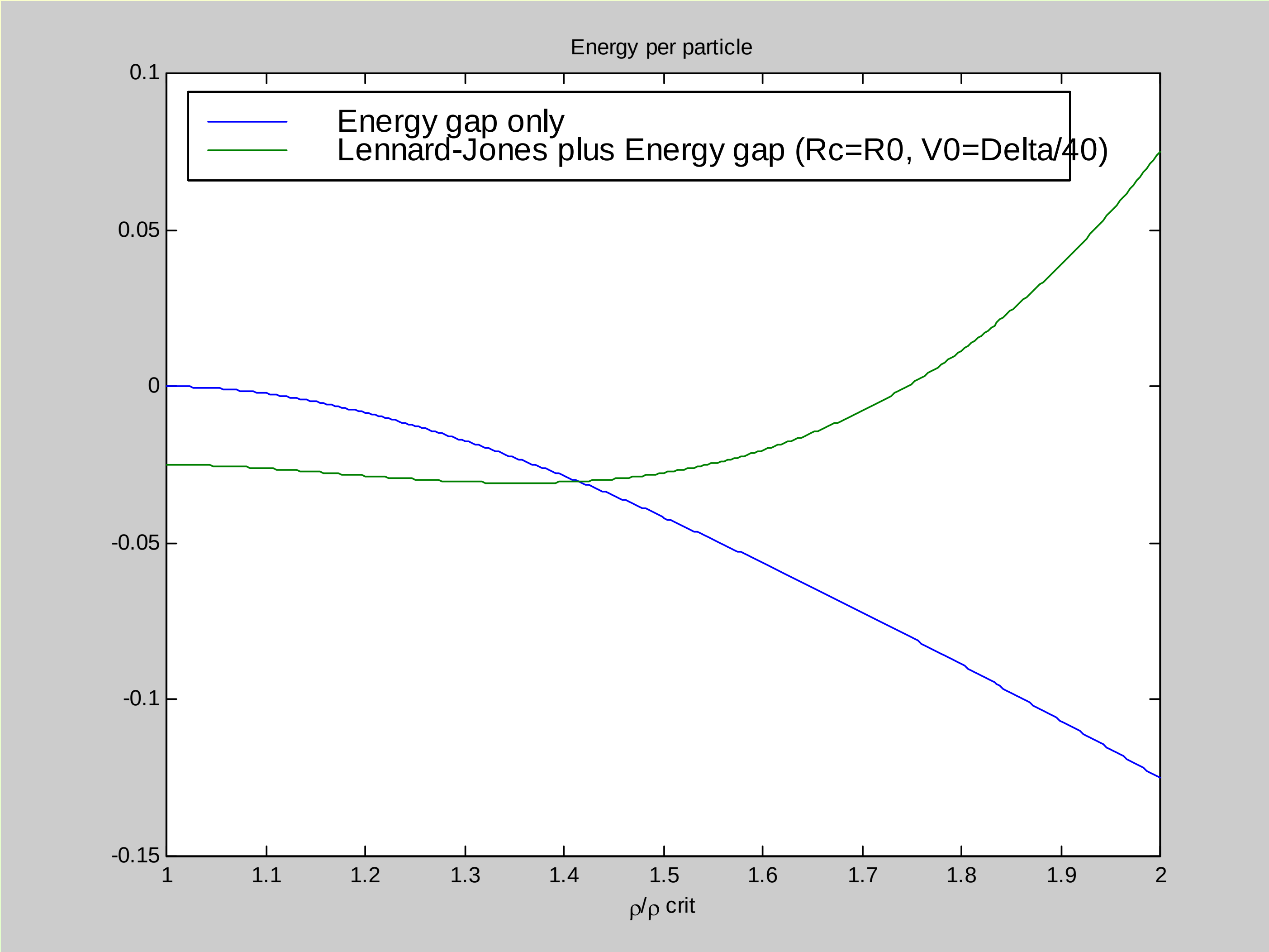
$$\rho > \rho_{\text{crit}} = \frac{\hbar\omega}{2\pi\mu_0^2}$$

Graphical representation of the energy





Energy as a function of density (example)



Important Remarks 1/2

Condensation occurs only if we have $\left\langle \sum_{i=1}^{N} \vec{\mu}_i \cdot \vec{E}(\vec{R}_i) \right\rangle_m = -N \sin\left(2\delta\right) \mu_0 \vec{\varepsilon}_1 \cdot \vec{E}(\vec{0})$,
and this happens only if the matter field has a modulation with period $\frac{4\pi}{k}$

The *ith* atom is in a configuration TOTALLY delocalized in space, in contrast with the particle-like character of its incoherent counterpart

The wave function of the single atoms in the coherent state is different from that of the lowest energy state without em condensate, since it contains a certain fraction of the excited state. This very important issue implies that, when matter interacts with external fields, it may exhibit unexpected behavior, a phenomenon referred to as *violation of asymptotic freedom*.

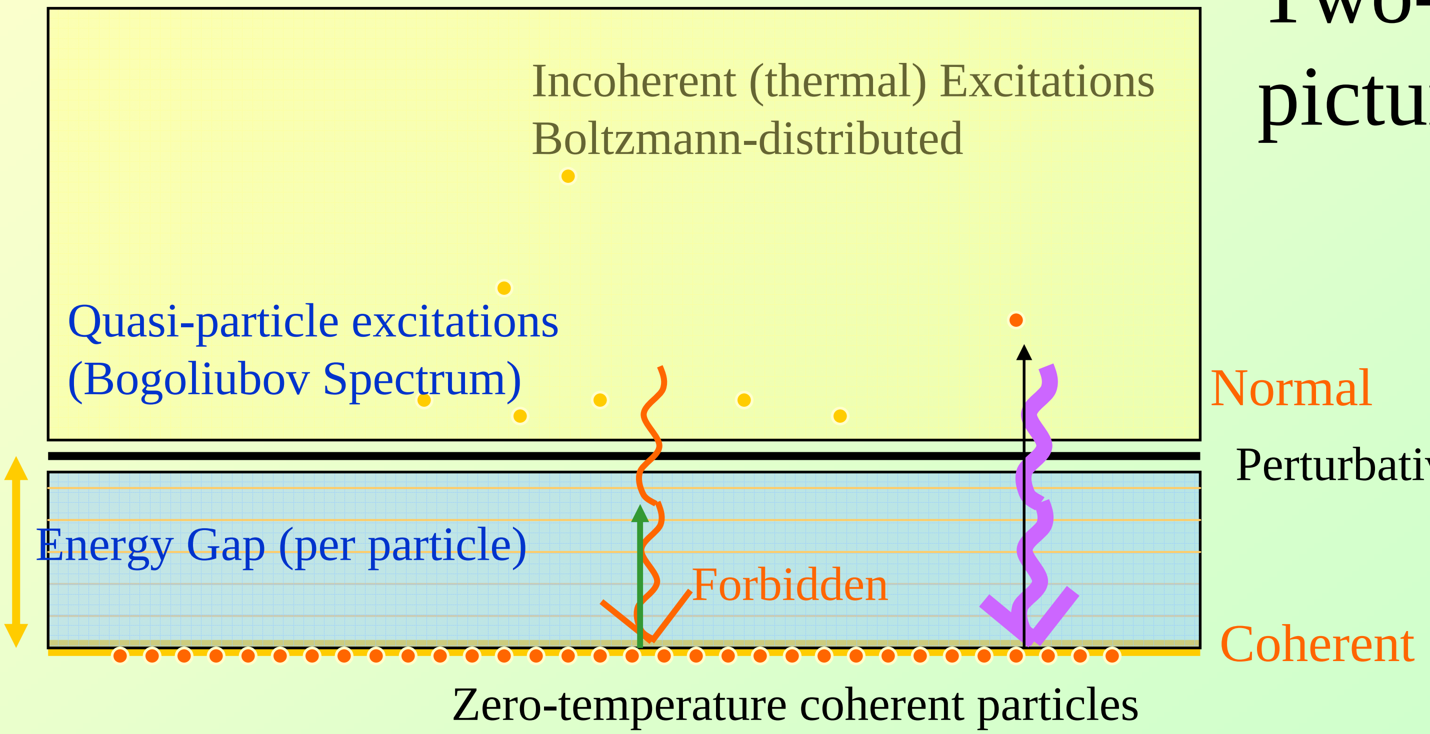
Important remarks 2/2

Simplified approach not applicable in this form to liquid water because of important dispersive contributions arising from excited levels of the water molecule (developed by G.Preparata and E.Del Giudice).

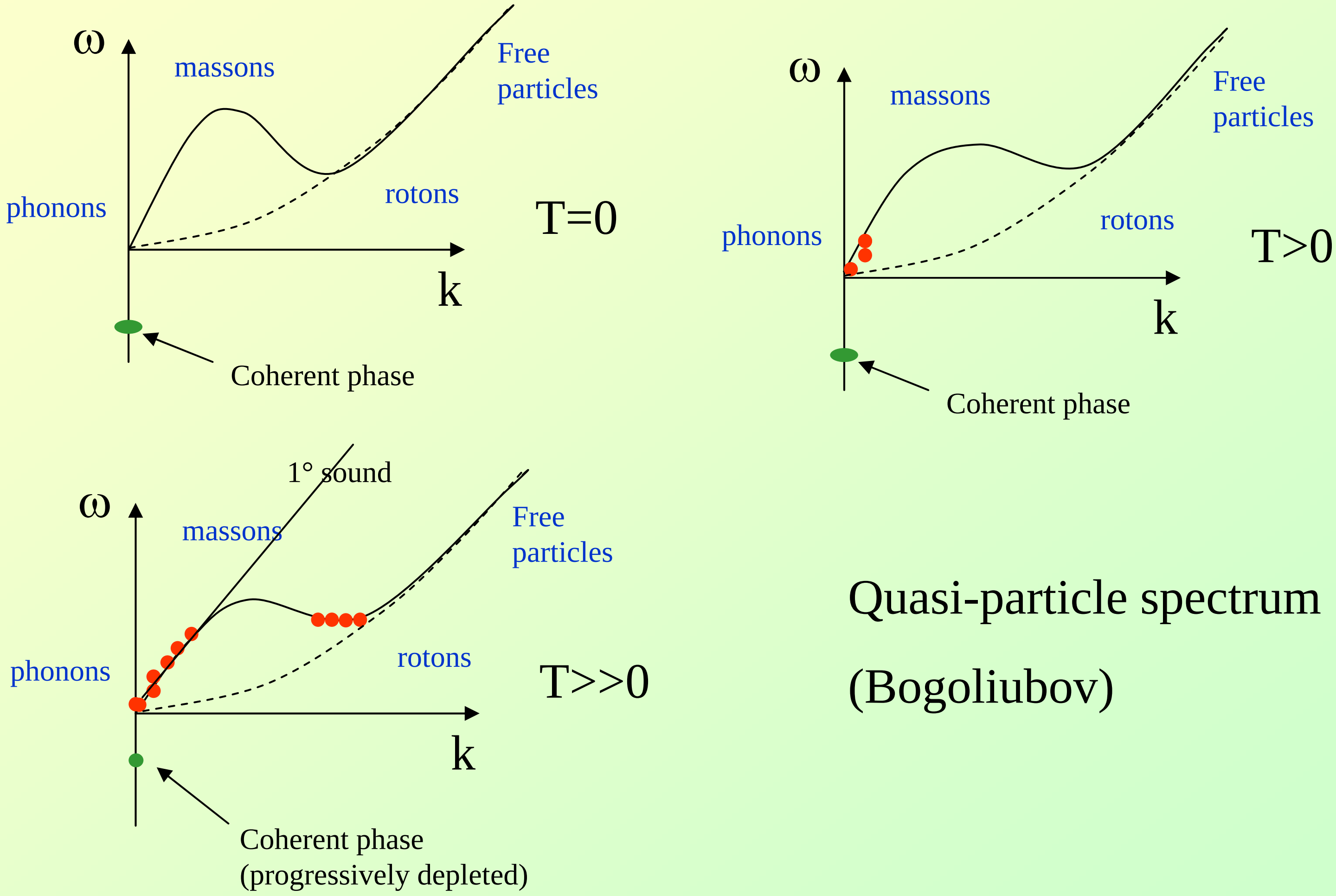
Complete revision of theory and numerical calculations is presently under way (myself and E.Del Giudice).

Energy of a coherent domain. The effect of temperature

Two-fluid picture



The effect of temperature

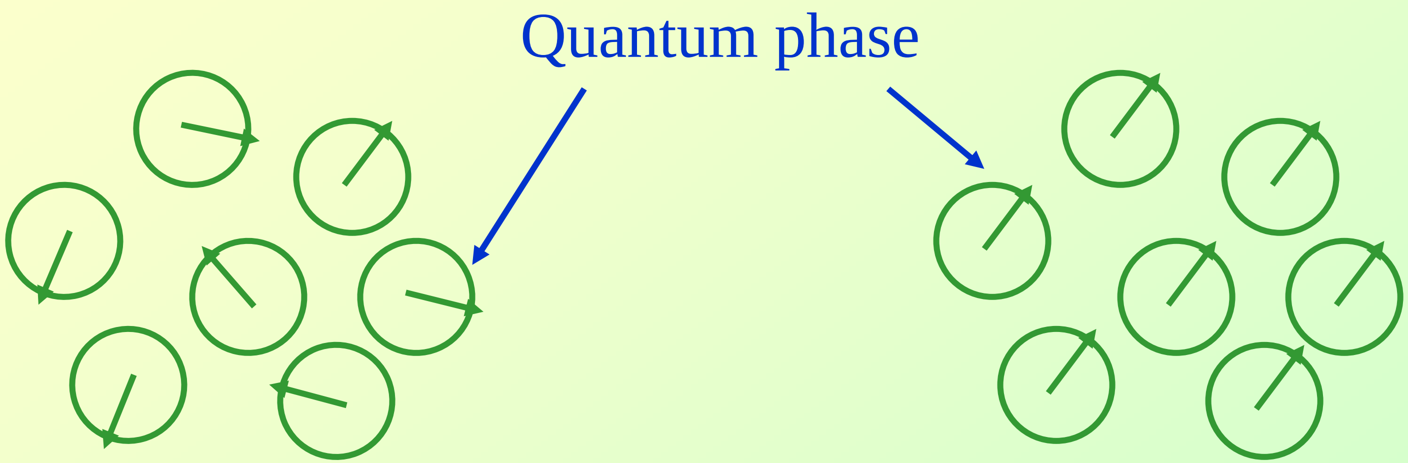


Quasi-particle spectrum

(Bogoliubov)

Quantum Coherent Interactions

N scatterers



Quantum incoherent

Quantum coherent

Cross section goes like N

Cross section goes like N^2

$N \sim 10^{23}$!!!

Theoretical consequences of quantum coherence in matter at finite temperature

- The existence of coherent configurations in matter implies the emergence of COHERENT SCATTERING

Incoherent scattering

$$A_{tot} = \sum_{j=1}^{N} A_1 e^{i\varphi_j} \longrightarrow \sigma_{tot} = N\sigma_1$$

Kinematics completely different!

Coherent scattering

$$A_{tot} = \sum_{j=1}^{N} A_1 \longrightarrow \sigma_{tot} = N^2\sigma_1$$

Coherent interactions: features

- Increased probability of interaction by orders of magnitude
- Different kinematics
- Energy exchange with up/downconversion
- Virtually no entropy generation
- Non-local interaction
- Seems adequate to the description of BIOLOGICAL PROCESSES

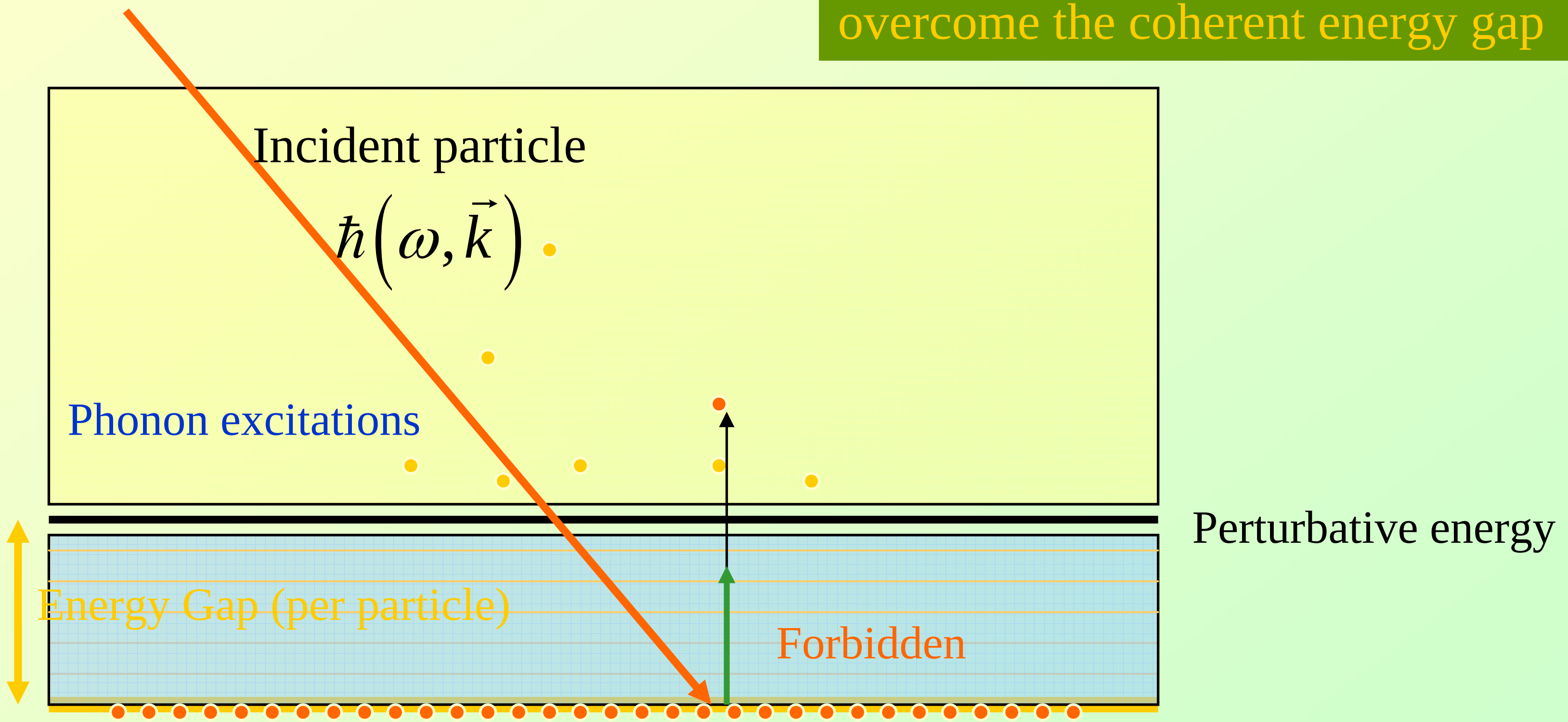
When

- Energy exchanged unable to overcome the energy gap

In most cases both coherent and incoherent interactions occur (e.g. Moessbauer effect) with relative balance depending on temperature

Conditions for coherent scattering

Incident particle must not be able to overcome the coherent energy gap



Technological issues

- Understanding of biological processes
  - New theoretical tool for biology
- Applications to quantum electronic devices (superradiant lasers)
  - Development of new devices exploiting energy up-conversion
  - Detectors of elusive particles (gravitons, neutrinos)
- Low-Energy Nuclear Reactions
  - Treatment of nuclear wastes
  - Generation of nuclear energy at ambient temperature
- Development of new branches in nanotechnology

Conclusions

- The generally accepted theory of condensed matter misses the contribution of the radiative field, that in particular circumstances cannot be neglected.
- Consideration of the radiative term brings to a potentially rich and powerful theroretical tool.
- New kinds of many-body, non-local interactions are possible even at high temperature (biology).

Although all that seems very promising, the theory is still in a preliminary stage and a large effort must be made before these ideas be universally accepted