

Is Cavitation Noise Governed by a Low-Dimensional Chaotic Attractor?

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Abstract. Using the example of periodically driven multi-bubble systems we demonstrate that it may be misleading to conclude from a successful low dimensional delay reconstruction that the underlying dynamical process is indeed governed by a low-dimensional attractor.

INTRODUCTION

Periodically driven nonlinear oscillators possess, even in the presence of chaos, a certain phase relation or coherence with respect to the driving force. As a result of this coherence, when chaos in an ensemble of oscillators is excited by a common periodic force, the cross-correlation between responses of two uncoupled or weakly coupled chaotic oscillators may not decay to zero but may become periodic for large time shifts. When this occurs, the average response of an ensemble of K such oscillators does not vanish but at large K becomes periodic. At intermediate values of K , time-delay reconstructions based on this averaged signal may appear to be noisy periodic, quasi-periodic or low-dimensional chaotic attractors and the true high-dimensionality of the signal is hidden in small scale oscillations of the measured quantity which are often experimentally not accessible. To give a concrete physical example of this general phenomenon, we consider an ensemble of micron-sized gas bubbles in water driven by an external sound field. These bubbles oscillate (chaotically) and the superposition of the sound waves emitted by the bubbles is measured with a hydrophone. The experimental data are compared with data obtained by numerical simulation.

NUMERICAL SIMULATION

Let us consider a system of K spherical gas bubbles, driven by a stationary sound field with wavelength large compared to the bubbles' radii and the distances between the bubbles. For simplicity, bubble-bubble interactions are neglected, but can be included with-

out difficulty. The radial motion of each bubble is assumed to obey the Keller-Miksis equation [1]

$$\left(1 - \frac{\dot{R}_k}{c}\right) R_k \ddot{R}_k + \frac{3}{2} \dot{R}_k^2 \left(1 - \frac{\dot{R}_k}{3c}\right) = \left(1 + \frac{\dot{R}_k}{c}\right) \frac{p_l}{\rho} + \frac{R_k}{\rho c} \frac{dp_l}{dt}, \quad (1)$$

$$p_l = \left(p_0 + \frac{2\sigma}{R_{0k}}\right) \left(\frac{R_{0k}}{R_k}\right)^{3\kappa} - p_0 - \frac{2\sigma}{R_k} - \frac{4\mu}{R_k} \dot{R}_k - p_a(t), \quad (2)$$

$$p_a(t) = P_a \cos(\omega t), \quad k = 1, \dots, K. \quad (3)$$

The dot denotes the time derivative. For air bubbles in water at 20° C with the polytropic exponent $\kappa = 1.4$ we use the following parameters: surface tension $\sigma = 0.0725$ N/m, liquid density $\rho = 998$ kg/m³, viscosity $\mu = 0.001$ Ns/m², ambient pressure $p_0 = 100$ kPa, sound velocity in the liquid $c = 1500$ m/s, a driving frequency $\omega = 2\pi \cdot 21$ kHz and a driving pressure $P = 150$ kPa. The numerical computations are performed with $K = 300$ bubbles with equilibrium radii R_{0k} uniformly distributed from 5 μm to 30 μm . The bubbles constitute a cloud given by a spatial Gaussian distribution with variance 1 cm. The hydrophone is placed 2.5 cm from the center of the bubble cluster.

Furthermore, we assume the motion of the liquid around the oscillating bubble to be spherically symmetric. In the vicinity of an oscillating bubble the incompressible liquid approximation is valid and the velocity field $w(r_k, t)$ may be written in the form

$$w = \frac{R_k^2 \dot{R}_k}{r_k^2}. \quad (4)$$

Here r_k is the radial coordinate and the origin of the local coordinate system coincides with the center of the bubble. To calculate the corresponding pressure field, the equation of liquid motion is used,

$$\rho \frac{\partial w}{\partial t} + \frac{\partial p_k}{\partial r_k} = 0, \quad (5)$$

where p_k is the pressure in the liquid emitted by the oscillating bubble. We have omitted the nonlinear convection term $w \partial w / \partial r_k$ in Eq. (5) because it is of the order of r_k^{-5} and therefore much smaller than the first term. Substitution of Eq. (4) into Eq. (5) and integration yields the following formula for the pressure:

$$p_k = \frac{\rho}{r_k} \frac{d}{dt} \left(R_k^2 \dot{R}_k \right) = \frac{\rho}{r_k} \left(R_k^2 \ddot{R}_k + 2R_k \dot{R}_k^2 \right). \quad (6)$$

It should be noted that Eq. (6) is only valid in the close vicinity of the bubble, when $r_k < c/\omega$. It is easy to show that for larger distances the following generalized formula may be derived:

$$p_k = \frac{\rho}{r_k} \frac{d}{dt} \left(R_k^2 \dot{R}_k \right) \Big|_{t-r_k/c}. \quad (7)$$

A low-pass filter is applied to p_k to take into account hydrophone characteristics (with cut-off frequency $f_0 = 400$ kHz)

$$\ddot{u}_k + 2\gamma\pi f_0 \dot{u}_k + 4\pi^2 f_0^2 u_k = p_k(t). \quad (8)$$

Finally, the hydrophone signal P_H is given by the superposition of the filtered pressure signals u_k from all bubbles, e. g.

$$P_H(t) = \sum_{k=1}^K u_k(t). \quad (9)$$

Figure 1 shows three-dimensional attractor reconstructions with delay time τ [2–4],

$$\mathbf{P}(t) = (P_H(t), P_H(t + \tau), P_H(t + 2\tau)) \quad (10)$$

from the experimentally measured and the numerically simulated hydrophone signal $P_H(t)$. Both reconstructions resemble noisy low-dimensional attractors. However, at least the numerically simulated hydrophone signal (Fig. 1(b)) is very high-dimensional, because for this example more than 70% of the $K = 300$ bubbles were found to oscillate chaotically. To resolve this high dimensionality of the time series from the hydrophone a huge amount of very high quality data would be necessary.

CONCLUSION

Delay reconstructions from time series generated by high-dimensional dynamics may yield objects (“attractors”) that are low-dimensional on experimentally observable

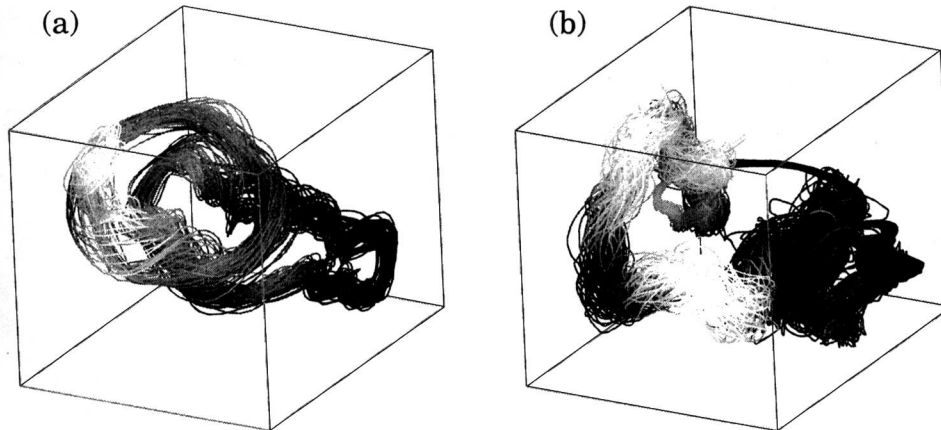


FIGURE 1. (a) Typical three-dimensional attractor reconstruction from the experimental hydrophone signal (see Refs. [5] and [6] for further details); (b) Attractor reconstruction from numerical simulations, which reproduces characteristic features of the attractor obtained from the experimental data.

medium and large scales. This effect occurs for large populations of driven (chaotic) oscillators with (very) high-dimensional attractors in the combined (product) state space of all oscillators. As a potential difficulty the apparent low dimensionality may be misinterpreted, for example in the analysis of predictability and controllability of the oscillator system. To demonstrate this phenomenon a set of $K = 300$ acoustically driven bubbles has been simulated. For simplicity the bubbles are assumed not to interact, but similar results are expected for more realistic simulations including interaction, surface oscillations, etc. Investigations with other oscillator systems showed that the main mechanism (phase relations between periodic drive and chaotic oscillators) is quite a general phenomenon [7].

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